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# LIQUID PAYLOAD MOMENT EQUATIONS

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER
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Murphy has developed linear-theory equations defining the moment exerted by a liquid payload on a spinning, coning projectile. This report re-writes Murphy's equations in a format more amenable to numerical calculations. The equations presented here are essentially those imbedded in our interactive computer program for generating liquid moment coefficients and eigenvalues.

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# I. INTRODUCTION

The flight of a spinning projectile with a liquid payload is sometimes briefer than planned. The liquid develops an intricate pattern of oscillations; if the frequency of one of these oscillations is close to one of the yawing frequencies of the shell, the yaw can grow unsuitably large. The projectile becomes unstable, its flight performance deteriorates rapidly and it fails to achieve its intended range.

Much work has gone into attempts to understand and describe mathematically what goes on between liquid payload and projectile. One of the workers in this field, C. H. Murphy, recently derived[1] a set of equations defining--under certain conditions--the transverse moment exerted by the fully spun-up liquid on the projectile. I have used these equations as the basis of an interactive computer program for generating liquid moment coefficients.

Murphy presented his equations in a form that may be intellectually satisfying but is several steps short of programming suitability. In particular, certain obvious tasks (taking indicated partial derivatives, obtaining closed-form expressions for integrals, etc.) were never undertaken. Murphy was aware of these omissions, and it was his hope that I would present the missing equations in a report. This is the report.

Sections II - V deal with how the computer program obtains the liquid moment coefficients. The remaining four sections discuss four alternative tasks that the program will perform upon request.

# II. THE TRANSVERSE LIQUID MOMENT COEFFICIENTS

In Reference 1, Murphy studies the response of the fully spun-up liquid to a projectile coning motion of constant frequency and exponentially changing magnitude:

$$\hat{\xi} = \hat{K}\, e^{s\phi} \tag{2.1}$$

where

$$s = (\epsilon + i)\,\tau \tag{2.2}$$

$$\phi = \dot{\phi} t \tag{2.3}$$

$$\dot{\phi} = \text{constant (positive) spin}$$

$$\hat{K} = \text{complex constant}$$

---

1. C. H. Murphy, "Angular Motion of a Spinning Projectile With a Viscous Liquid Payload," Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Memorandum Report ARBRL-MR-03194, August 1982. (AD A118676) (See also Journal of Guidance, Control, and Dynamics, Vol. 6, July-August 1983, pp. 280-286.)

and where $\hat{\xi}$ is the complex yaw in an aeroballistic non-rolling coordinate system.

The parameters in Murphy's complex variable s have the following interpretation:

$\tau$ = (coning rate)/$\dot{\phi}$, the nondimensionalized coning frequency*

$\varepsilon$ = (yaw damping rate)/(coning rate), the nondimensionalized yaw damping ($\varepsilon < 0$) or yaw growth ($\varepsilon > 0$) rate. (Thus $\varepsilon = 0$ is the important special case of constant-amplitude coning motion.)

Murphy considers the liquid payload to be confined to a right circular cylinder of diameter 2a and height 2c. The axis of this cylinder is collinear with the projectile's principal axis (the spin axis); the center of the cylinder is located a distance h forward of the projectile's center of mass. The offset h will be taken as zero in the body of this report; nonzero h is relegated to Appendix E.

The cylinder may be fully or only partially filled with liquid. In either case, the liquid is assumed to be fully spun-up. For the partially-filled cylinder, it is assumed that the liquid has formed a cylinder of diameter 2a with an air core of diameter 2b. (Murphy also considers the case of a fully-filled cylinder with a central rod. That is, the free surface r = b is replaced by an inner lateral surface r = d, where 2d is the rod diameter. This central rod case is discussed in Appendix F.)

Murphy introduces two real liquid moment coefficients, $C_{LSM}$ and $C_{LIM}$, defined by the moment equation

$$M_{L\hat{Y}} + i\, M_{L\hat{Z}} = m_L a^2 \dot{\phi}^2\, \tau\, C_{LM}\, \hat{\xi} \tag{2.4}$$

where

$M_{L\hat{Y}} + i\, M_{L\hat{Z}}$ = the transverse liquid moment in the aeroballistic system

*Other notation appears occasionally in the literature. Some authors have used the symbol $\tau$ to denote the <u>complex</u> frequency that would be written here as $-is$.

$$m_L = 2\pi a^2 c \rho_L \tag{2.5}$$

= the mass of the liquid in a fully-filled cylinder

$\rho_L$ = liquid density

$$C_{LM} = C_{LSM} + i\, C_{LIM}. \tag{2.6}$$

The $C_{LSM}$ term in Eq. (2.4) is the liquid side moment, a moment causing rotation out of the transverse plane. The $C_{LIM}$ term in Eq. (2.4) is the liquid in-plane moment, a moment causing rotation in the transverse plane.

The bulk of Reference 1 is devoted to deriving an expression for $C_{LM}$. This expression involves earth-fixed cylindrical coordinates x, r, θ and four complex perturbation variables defined in Eqs. (3.10-3.13) of Reference 1:

$p_s$ = nondimensional pressure perturbation,

$u_s$, $v_s$, $w_s$ = nondimensional velocity perturbation components in the x, r, θ directions, respectively.*

Each of these variables is a function of x and r and each - in Murphy's linear theory - is linear in $\hat{K}$.

Murphy makes the assumption that each of the four variables can be expressed as the sum of an inviscid and a viscous term:

$$p_s\,(r,x) = p_{si}\,(r,x) + p_{sv}\,(r,x) \tag{2.7}$$

and similarly for the velocity perturbations.

---

**The association of (u, v, w) with the ordering (x, r, θ) seems logically sound and reflects Murphy's adherence to a standard reference on letter symbols[2] and to the notation he has used in over 30 publications on symmetric missile dynamics. However, other writers on liquid-filled shell associate (u, v, w) with (r, θ, x) [or in their notation (r, θ, z)]; still others associate (u, v, w) with rectangular coordinates. This nonuniformity is only mildly amusing.*

2. *"American Standard Letter Symbols for Aeronautical Sciences," ASA Y10.7-1954, published by The American Society of Mechanical Engineers, October 1954.*

Making a few additional assumptions and adhering rigidly to linear theory, Murphy derives the following relations:

$$C_{LM} = (C_{LM})_{p\ell} + (C_{LM})_{pe} + (C_{LM})_{v\ell} + (C_{LM})_{ve} \tag{2.8}$$

where

$$(C_{LM})_{p\ell} = \frac{i}{2ac\tau} \int_{-c}^{c} \left[ \hat{K}^{-1} x\, p_s(a,x) - x^2/a \right] dx \tag{2.9}$$

$$(C_{LM})_{pe} = \frac{-i}{a^2 c\tau} \int_{b}^{a} \left[ \hat{K}^{-1} r^2\, p_{si}(r,c) - cr^3/a^2 \right] dr \tag{2.10}$$

$$(C_{LM})_{v\ell} = (2\hat{K}\ \mathrm{Re}\ c\tau)^{-1} \int_{-c}^{c} \left[ x \frac{\partial w_{sv}}{\partial r} + i\, a \frac{\partial u_{sv}}{\partial r} \right]_{r=a} dx \tag{2.11}$$

$$(C_{LM})_{ve} = (\hat{K}\ \mathrm{Re}\ a\tau)^{-1} \int_{b}^{a} \left[ \frac{\partial (w_{sv} - i\, v_{sv})}{\partial x} \right]_{x=c} r\,dr \tag{2.12}$$

and where $\mathrm{Re} = \dot{\phi} a^2/\nu$, the Reynolds number.

The subscripts on $C_{LM}$ have the following meaning:

| Subscript | Denotes liquid moment due to: |
|---|---|
| $p\ell$ | pressure on the cylinder's lateral wall |
| pe | pressure on the end walls |
| $v\ell$ | viscous shear on the lateral wall |
| ve | viscous shear on the end walls |

The next three sections will be devoted to converting Eqs. (2.9 - 2.12) to programmable form.

## III. THE FUNCTIONS $X_k(x)$

Murphy derives expressions and conditions for the perturbation variables that involve complex functions of r: $R_k(r)$, and complex functions of x: $X_k(x)$. In this section, we define and discuss $X_k(x)$ and some related constants.

The subscript k is the so-called axial wave number, the number of nodes in the liquid's axial wave pattern. The liquid has three wave numbers, (k, n, m): non-negative integers associated with the axial, radial, and azimuthal wave patterns, respectively. Since the functions in Eqs. (2.9-2.12) are evaluated for m = 1, only two wave numbers, k and n, concern us here.

The axial wave number k is either zero or an odd, positive integer. A term from the k = 0 mode is always present in $C_{LM}$ to satisfy the inviscid end-wall boundary condition. However, Murphy gives this term explicitly, circumventing the need for a k = 0 subscript in this context. An additional term for the zero mode is required when the center of the cylinder is offset axially a distance h from the center of mass of the projectile; this case has been banished to Appendix E. Hence, k will not assume the value 0 in this report:

$$k = 1, 3, 5, \ldots \tag{3.1}$$

Before we discuss $X_k(x)$, we introduce the complex $\lambda_k = \lambda_k$ (s, c/a, Re), which can be computed as the solution of the equation

$$1 + \lambda_k \delta_c \tan \lambda_k = 0 \tag{3.2}$$

where

$$\delta_c = \frac{-(a/c)\ \delta_a}{2\sqrt{(1 + is)}} \left[ \frac{1 - is}{\sqrt{3 + is}} + i \left( \frac{3 + is}{\sqrt{1 - is}} \right) \right] \tag{3.3}$$

$$\delta_a = \frac{1 + i}{\sqrt{2(1 + is)\ Re}} \tag{3.4}$$

We are not concerned here with the derivation of these and subsequent equations; we are merely trying to show what must be done by a computer program to obtain results. One of the first tasks for our program is the solving of Eq. (3.2) by an iterative process (the Newton method). A good first estimate (for small $|\delta_c|$, that is, for large Re) is the approximation

$$\lambda_k \cong \frac{\pi k}{2\ (1 - \delta_c)} \ . \tag{3.5}$$

Murphy's complex functions $X_k(x)$ have the form:

$$X_k(x) = \sin\ (\lambda_k\ x/c)\ . \tag{3.6}$$

Next, we introduce three sets of constants: $b_k$, $b_{jk}$, $a_k$, all dependent on $X_k(c)$. The first two are needed to compute $a_k$ and $a_k$ is needed to produce the functions $R_k(r)$ of the next section. We have

$$
\left.
\begin{aligned}
b_k &\equiv c^{-2} \int_{-c}^{c} \bar{X}_k(x)\; x\, dx \\
&= 2\, \bar{X}_k(c) \left[ \bar{\delta}_c + \bar{\lambda}_k^{-2} \right]
\end{aligned}
\right\}
\tag{3.7}
$$

$$
\left.
\begin{aligned}
b_{jk} &\equiv c^{-1} \int_{-c}^{c} \bar{X}_j(x)\; X_k\,(x)\; dx \\
&= \frac{2\, \bar{X}_j(c)\; X_k(c) \left[ \lambda_k^{\,2}\, \delta_c - (\bar{\lambda}_j)^2\, \bar{\delta}_c \right]}{\lambda_k^{\,2} - (\bar{\lambda}_j)^2}
\end{aligned}
\right\}
\tag{3.8}
$$

where j, like k, takes on the values 1, 3, 5, ...

The complex constants $a_k$ are obtained as the solution set of the system

$$
\sum_{k=1}^{N} b_{jk}\, a_k = b_j, \qquad j = 1, 3, 5, \ldots N \tag{3.9}
$$

where the value of N is somewhat arbitrary. In practice, N in our program is set at 29; this allows k to take on fifteen values. In general, solving system (3.9) involves inverting an (N + 1)/2 by (N + 1)/2 complex matrix.

It should be noted that the $a_k$'s determined in this manner are precisely the least squares coefficients when x/c is approximated by a truncated series in $X_k(x)$:

$$
\frac{x}{c} \mathrel{\hat{=}} \sum_{k=1}^{N} a_k\, X_k(x) \tag{3.10}
$$

In particular

$$
\sum_{k=1}^{N} a_k\, X_k(c) \mathrel{\hat{=}} 1 \tag{3.11}
$$

Of less use to us, but still interesting, is the fact that

$$
\sum_{k=1}^{N} a_k\, \bar{b}_k \mathrel{\hat{=}} 2/3\,. \tag{3.12}
$$

For the special case of infinite Reynolds number, we have:

$$\left.\begin{aligned}
&\delta_a = \delta_c = 0 \\
&\lambda_k = \pi k/2 \\
&X_k(c) = (-1)^{(k-1)/2} \\
&b_{jk} = \begin{cases} 0 & \text{if } j \neq k \\ 1 & \text{if } j = k \end{cases} \\
&b_k = a_k = \left[8/(\pi k)^2\right](-1)^{(k-1)/2}
\end{aligned}\right\} Re^{-1} \equiv 0 \tag{3.13}$$

## IV. THE FUNCTIONS $R_k(r)$

Murphy's complex functions $R_k(r)$ have the form

$$R_k(r) = a_k\left[E_k\ J_1\ (A_k) + F_k\ Y_1\ (A_k)\right] \tag{4.1}$$

where

$$A_k = A_k(r) = \hat{\lambda}_k\ r/c \tag{4.2}$$

$$\hat{\lambda}_k = \frac{[(3 + is)\ (1 - is)]^{1/2}\ \lambda_k}{1 + is} \tag{4.3}$$

$J_i$ is the Bessel function of the first kind of order i (we will only be concerned with i = 0 and 1)

$Y_i$ is the Bessel function of the second kind of order i

and where $E_k(s)$ and $F_k(s)$ are complex functions defined by Eq. (4.1). This definition sheds little light on how $E_k$ and $F_k$ can be evaluated. By determining conditions that $R_k$ must satisfy at the boundaries r = a and r = b, Murphy obtains two equations in the two unknowns $E_k$ and $F_k$:

$$\left.\begin{aligned} c_{11}\, E_k + c_{12}\, F_k &= c_1 \\ c_{21}\, E_k + c_{22}\, F_k &= c_2 \end{aligned}\right\} \qquad (4.4)$$

where, setting $\tilde{\delta}_a = \delta_a (1 - \delta_a)^{-1}$, we have*

$$\left.\begin{aligned} c_{11} = {} & [(1 + 2\, \tilde{\delta}_a)\, (1 - is) + (1 + is)\, A_{ka}{}^2\, \tilde{\delta}_a ]\, J_{1a} \\ & + [1 + is - (1 - is)\, \tilde{\delta}_a]\, A_{ka}\, J_{0a} \end{aligned}\right\} \qquad (4.5)$$

$c_{12}$ = {form identical to $c_{11}$ with J replaced by Y}

$$c_{21} = [2\, (1 + is) + (3 + is)s^2]\, J_{1b} - (1 + is)\, A_{kb}\, J_{0b} \qquad (4.6)$$

$c_{22}$ = {form identical to $c_{21}$ with J replaced by Y}

$$c_1 = 2is\, (1 + is)\, (3 + is) \qquad (4.7)$$

$$c_2 = (b/a)s^2\, (1 + is)^2\, (3 + is) \qquad (4.8)$$

and where

$$\left.\begin{aligned} A_{ka} &= A_k(a) = \hat{\lambda}_k\, a/c \\ J_{0a} &= J_0\, (A_{ka}) \\ J_{1a} &= J_1\, (A_{ka}) \\ Y_{0a} &= Y_0\, (A_{ka}) \\ Y_{1a} &= Y_1\, (A_{ka}) \end{aligned}\right\} \qquad (4.9)$$

and similarly for $r = b$.

* *In Reference 1, Murphy approximates $\tilde{\delta}_a$ by $\delta_a$ in $c_{11}$ and $c_{12}$. This introduces an error that is negligible except at very low Reynolds numbers.*

Solving the system (4.4), we have

$$E_k = \frac{c_1 - (c_{12}/c_{22})\ c_2}{G_k} \tag{4.10}$$

$$F_k = \frac{(c_{11}/c_{22})\ c_2 - (c_{21}/c_{22})\ c_1}{G_k} \tag{4.11}$$

where

$$G_k = c_{11} - (c_{12}/c_{22})\ c_{21} \tag{4.12}$$

The special situation $G_k(s) = 0$ will be discussed in Section VII on eigenvalues.

Note that for $b = 0$ (that is, for the usual case of a fully-filled cylinder), $c_{22}$ is infinite, so that Eqs. (4.10-4.12) reduce to

$$\left.\begin{aligned} E_k &= c_1/c_{11} \\ F_k &= 0 \\ G_k &= c_{11} \end{aligned}\right\} \qquad b \equiv 0 \quad . \tag{4.13}$$

Finally, we note that

$$r\ R_k'\ (r) = a_k\ A_k \left[E_k\ J_0\ (A_k) + F_k\ Y_0\ (A_k)\right] - R_k \tag{4.14}$$

$$r^2\ R_k''\ (r) = (1 - A_k^2)\ R_k - r\ R_k' \tag{4.15}$$

where the primes denote differentiation with respect to r.

## V. THE TRANSVERSE LIQUID MOMENT COEFFICIENT EQUATIONS

Expressions for the perturbation variables in terms of $X_k(x)$ and $R_k(r)$ are given in Appendices A, B, and C. These expressions are not themselves part of the computer program, but they are useful in the derivation of the programmed equations. The nature of this derivation is outlined in Appendix D.

In the body of this report, as we have indicated, we are interested mainly in presenting the programmed equations for $C_{LSM}$ and $C_{LIM}$. We can now do so:

$$(C_{LM})_{p\ell} = \frac{i\ (c/a)^2}{\tau}\left[\frac{is\ (2 + is)}{3} - \frac{T_1}{2} + \delta_a T_5\right] \tag{5.1}$$

$$(C_{LM})_{pe} = \frac{i}{\tau}\left\{\left(\frac{c}{a}\right)^2 \left[T_3 - \left(\frac{b}{a}\right) T_{3b}\right] - \frac{is\ (2 + is)}{4}\left(1 - \frac{b^4}{a^4}\right)\right\} \tag{5.2}$$

$$(C_{LM})_{v\ell} = -\frac{i\ \delta_a}{\tau}\left[T_4 + \frac{(c/a)^2\ (1 + is)\ T_5}{2}\right] \tag{5.3}$$

$$(C_{LM})_{ve} = \frac{(c/a)\,\delta_a}{\tau}\left(\frac{1 + is}{1 - is}\right)^{1/2}\left[T_4 - (b/a)\ T_{4b} - 2\ is\ (1 + is)\left(1 - \frac{b^2}{a^2}\right)\right] \tag{5.4}$$

where

$$T_1 = \sum R_k(a) \cdot \bar{b}_k \tag{5.5}$$

$$T_2 = \sum a\ R_k'(a) \cdot \bar{b}_k \tag{5.6}$$

$$T_3 = \sum [R_k(a) - a\ R_k'(a)] \cdot \hat{\lambda}_k^{-2} \cdot X_k(c) \tag{5.7}$$

$$T_{3b} = \sum [R_k(b) - b\ R_k'(b)] \cdot \hat{\lambda}_k^{-2} \cdot X_k(c) \tag{5.8}$$

$$T_4 = \sum R_k(a) \cdot X_k(c) \tag{5.9}$$

$$T_{4b} = \sum R_k(b) \cdot X_k(c) \tag{5.10}$$

$$T_5 = \frac{1}{1 - is}\left[\frac{(1 + is)\ T_1 + 2\ T_2}{3 + is} - \frac{4\ is\ (1 + is)}{3}\right] \tag{5.11}$$

and where the summations are for $k = 1, 3, 5, \ldots N$.

To produce tabulated values and plots of $C_{LSM}$ and $C_{LIM}$ versus $\tau$, the computer program carries out the following instructions:

1. Accept the required input:
   Re, b/a, c/a, $\varepsilon$ and the $\tau$ range of interest.

2. Divide the $\tau$ range into, say, several hundred equally-spaced points $\tau_1, \tau_2, \ldots$

3. Perform steps 4 - 10 below <u>for each $\tau_i$</u>.

   4. For the current s value, compute $\delta_a$, $\delta_c$ [Eqs. (3.4, 3.3)] and $c_1$, $c_2$ [Eqs. (4.7, 4.8)].

   5. Perform steps 5a - 5c below for k = 1, 3, 5,...N (=29):

      5a. Iterate on Eq. (3.2) to obtain $\lambda_k$.

      5b. Compute $X_k(c)$ and $b_k$ [Eqs. (3.6, 3.7)].

      5c. For j = 1, 3, 5,...N, compute $b_{jk}$ [Eq. (3.8)].

   6. Solve the system (3.9) for all the $a_k$'s.

   7. Perform steps 7a - 7g below for k = 1, 3, 5,...N (=29):

      7a. Compute $\dot{\lambda}_k$ [Eq. (4.3)].

      7b. Compute $A_{ka}$, $A_{kb}$ [Eq. (4.9)].

      7c. Compute the complex Bessel functions at r = a and b:
      $J_{0a}$, $J_{1a}$, $Y_{0a}$, $Y_{1a}$, $J_{0b}$, $J_{1b}$, $Y_{0b}$, $Y_{1b}$.

      7d. Compute $c_{11}$, $c_{12}$, $c_{21}$, $c_{22}$ [Eqs. (4.5, 4.6)].

      7e. Compute $G_k$, $E_k$, and $F_k$ [Eqs. (4.12, 4.10, 4.11)].

7f. Compute $R_k(a)$ and $R_k(b)$ [Eq. (4.1)].

7g. Compute $aR_k'(a)$ and $bR_k'(b)$ [Eq. (4.14)].

8. Form the sums $T_i$ [Eqs. (5.5 - 5.11)].

9. Compute $(C_{LM})_{p\ell,\ pe,\ v\ell,\ ve}$ [Eqs. (5.1 - 5.4)].

10. Compute, print, and store $C_{LSM} = R\{C_{LM}\}$ and $C_{LIM} = I\{C_{LM}\}$.

11. Plot $C_{LSM}$ and $C_{LIM}$ versus $\tau$.

I am occasionally bemused by the fact that the entire process above takes only a fraction of a second per point on our computer (a Digital Equipment Corporation VAX-11/780) while it simultaneously keeps a dozen or so other terminal users happy.

## VI. THE LIQUID ROLL MOMENT COEFFICIENT

As mentioned, our program has the ability to perform tasks in addition to, or instead of, computing $C_{LSM}$ and $C_{LIM}$ versus $\tau$. The simplest of these optional chores is the computation of the liquid roll moment coefficient, $C_{LRM}$, versus $\tau$. $C_{LRM}$ is defined by the moment equation

$$M_{L\hat{X}} = m_L a^2 \dot{\phi}^2 \tau |\tilde{\xi}|^2 C_{LRM} \tag{6.1}$$

where $M_{L\hat{X}}$ is the axial liquid moment (compare Eq. (2.4)).

In Reference 3, Murphy shows that for his linear fluid mechanics assumptions $C_{LRM}$ is very nearly (or exactly, when $\varepsilon = 0$) the negative of $C_{LSM}$:

$$C_{LRM} = -C_{LSM} + (\tau\varepsilon/2)\,[1 - (4/3)\,(c/a)^2]\ . \tag{6.2}$$

Thus, the computation of $C_{LRM}$ from $C_{LSM}$ is trivial and we move on at once to more interesting options.

3. *C. H. Murphy, "Liquid Payload Roll Moment Induced by a Spinning and Coning Projectile," U.S. Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Technical Report ARBRL-TR-02521, September 1983. (AD A133681) (See also AIAA Paper 83-2142, August 1983.)*

## VII. EIGENVALUES

For a given Re, a, b, c, and k, the complex variable $G_k$ of Eq. (4.12) is a function of s. For some set of values $s = \{s_{kn}\}$, $G_k$ will be zero:

$$G_k (s_{kn}) = 0 \quad . \tag{7.1}$$

These values of s are by definition the eigenvalues of the system.

Index n (the radial wave number mentioned in Section III) is a positive integer and k, as before, takes on the values 1, 3, 5,... In theory, there are an infinite number of eigenvalues, but their practical importance (that is, their effect on a projectile's performance) usually decreases as k and n increase.

One of the options in the program developed from Murphy's equations is the determination of the more important of these eigenvalues. This is done by solving Eq. (7.1) by the Newton iterative method, in which

$$(s_{kn})_{i+1} = (s_{kn})_i - \left[\frac{G_k(s)}{G_k'(s)}\right]_{s = (s_{kn})_i} \tag{7.2}$$

where the prime denotes differentiation with respect to the complex variable s.

The value of k must be specified; the value of n is implicit in the required initial estimate $(s_{kn})_0$. For a given c/a and k, the program determines six initial estimates (n=1,2,.....6) by interpolating in a stored table. This table contains a set of real eigenvalues $\tau_{kn}$ ($Re^{-1} = 0$, $b = 0$) versus c/(ak) for each of the six n values. The stored range of $\tau_{kn}$ is -1 to 1; a portion of this table, 0 to 0.6, is given here as Table 1. From these six initial estimates (the value "six" is arbitrary), the program attempts to find the first six eigenvalues:

$$s_{k1},\ s_{k2},\ldots s_{k6}$$

by iterating on Eq. (7.2). Success here depends on the adequacy of the estimates from the stored table. These estimates necessarily become poorer as the given Re decreases and/or b increases. Hence, it is possible (but rare, in our experience) for the iterative process to converge to a "wrong" eigenvalue; that is, to an eigenvalue associated with some other n than the one indicated by the program. (This problem could be avoided - or at least passed on to the user - by a program change that would require the user to input the six eigenvalue estimates. So far, this has not been necessary.)

Table 1. Eigenvalues $\tau_{kn}$ for $Re^{-1} = 0$, b = 0.

| $\tau_{kn}$ | c/(ak) n = 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 0 | .9949 | .4780 | .3103 | .2291 | .1814 | .1501 |
| .01 | 1.0064 | .4842 | .3144 | .2322 | .1838 | .1521 |
| .02 | 1.0180 | .4904 | .3186 | .2353 | .1863 | .1542 |
| .03 | 1.0297 | .4968 | .3228 | .2384 | .1888 | .1562 |
| .04 | 1.0417 | .5032 | .3271 | .2416 | .1914 | .1583 |
| .05 | 1.0538 | .5098 | .3315 | .2449 | .1939 | .1605 |
| .06 | 1.0661 | .5165 | .3360 | .2482 | .1966 | .1627 |
| .07 | 1.0786 | .5233 | .3405 | .2516 | .1993 | .1649 |
| .08 | 1.0913 | .5302 | .3451 | .2550 | .2020 | .1672 |
| .09 | 1.1042 | .5372 | .3498 | .2585 | .2048 | .1695 |
| .10 | 1.1173 | .5443 | .3546 | .2621 | .2077 | .1719 |
| .11 | 1.1306 | .5516 | .3595 | .2657 | .2106 | .1743 |
| .12 | 1.1442 | .5591 | .3645 | .2695 | .2135 | .1767 |
| .13 | 1.1579 | .5666 | .3695 | .2732 | .2165 | .1792 |
| .14 | 1.1720 | .5743 | .3747 | .2771 | .2196 | .1818 |
| .15 | 1.1862 | .5822 | .3800 | .2810 | .2227 | .1844 |
| .16 | 1.2008 | .5902 | .3854 | .2851 | .2259 | .1870 |
| .17 | 1.2156 | .5983 | .3908 | .2892 | .2292 | .1898 |
| .18 | 1.2307 | .6067 | .3965 | .2934 | .2326 | .1925 |
| .19 | 1.2460 | .6152 | .4022 | .2977 | .2360 | .1954 |
| .20 | 1.2617 | .6239 | .4080 | .3020 | .2395 | .1983 |
| .21 | 1.2777 | .6327 | .4140 | .3065 | .2430 | .2012 |
| .22 | 1.2940 | .6418 | .4201 | .3111 | .2467 | .2043 |
| .23 | 1.3107 | .6511 | .4264 | .3158 | .2504 | .2074 |
| .24 | 1.3277 | .6605 | .4328 | .3206 | .2543 | .2105 |
| .25 | 1.3450 | .6702 | .4394 | .3255 | .2582 | .2138 |
| .26 | 1.3628 | .6801 | .4461 | .3305 | .2622 | .2171 |
| .27 | 1.3809 | .6903 | .4529 | .3357 | .2663 | .2206 |
| .28 | 1.3995 | .7007 | .4600 | .3410 | .2705 | .2241 |
| .29 | 1.4185 | .7113 | .4672 | .3464 | .2748 | .2277 |
| .30 | 1.4379 | .7222 | .4746 | .3520 | .2793 | .2313 |
| .32 | 1.4783 | .7449 | .4900 | .3635 | .2885 | .2390 |
| .34 | 1.5206 | .7687 | .5062 | .3757 | .2983 | .2471 |
| .36 | 1.5652 | .7939 | .5234 | .3887 | .3086 | .2557 |
| .38 | 1.6123 | .8204 | .5416 | .4023 | .3195 | .2648 |
| .40 | 1.6621 | .8486 | .5608 | .4168 | .3311 | .2745 |
| .42 | 1.7149 | .8784 | .5813 | .4323 | .3435 | .2848 |
| .44 | 1.7710 | .9102 | .6031 | .4488 | .3567 | .2957 |
| .46 | 1.8308 | .9441 | .6264 | .4664 | .3708 | .3075 |
| .48 | 1.8947 | .9803 | .6514 | .4853 | .3859 | .3201 |
| .50 | 1.9632 | 1.0192 | .6782 | .5056 | .4022 | .3337 |
| .52 | 2.0369 | 1.0610 | .7070 | .5275 | .4198 | .3483 |
| .54 | 2.1164 | 1.1061 | .7382 | .5511 | .4388 | .3642 |
| .56 | 2.2027 | 1.1549 | .7721 | .5769 | .4595 | .3814 |
| .58 | 2.2965 | 1.2081 | .8090 | .6049 | .4820 | .4002 |
| .60 | 2.3991 | 1.2661 | .8493 | .6356 | .5067 | .4208 |

The computation of $G_k'(s)$ deserves a comment or two. It is barely possible to derive an exact expression for $G_k'$; in an atypical fit of zeal, I have done so. The difficulty lies in the fact that everything but the temperature in Newark depends on s. It was necessary to derive expressions for $\delta_a'$, $\delta_c'$, $\lambda_k'$, $\hat{\lambda}_k'$, $J_{0a}'$, etc., etc., in a tortuous chain culminating in $G_k'$ as a function of the $C_{ij}$'s and their derivatives. I won't inflict these equations on the reader for two reasons: (1) they would take up a lot of space and time; (2) they aren't really necessary. The approximation

$$G_k'(s) = \frac{G_k(s + \Delta s/2) - G_k(s - \Delta s/2)}{\Delta s} \tag{7.3}$$

should be more than adequate if the increment $\Delta s$ is chosen with a little care. This approximation eliminates the myriad opportunities to err that arise in deriving and coding the exact expression.

## VIII. THE EFFECT OF THE LIQUID MOMENT ON THE DAMPING

In Section V, we showed how our program determines $C_{LSM}(\tau)$ and $C_{LIM}(\tau)$ for a fixed $\varepsilon$. But $\varepsilon$ and $\tau$ are not really independent; they are related by the yaw equation. In Reference 1, Murphy derives the following relationship (in slightly different notation):

$$\varepsilon(\tau) = \frac{B_1\ C_{LSM}(\varepsilon,\tau)}{2\ B_2\ [1 + B_1\ C_{LIM}(\varepsilon,\tau)] - 1} + \varepsilon_A \tag{8.1}$$

where

$B_1 = m_L a^2/I_x$

$B_2 = 1 - (4s_g)^{-1}$

$I_x$ = axial moment of inertia of the projectile

$s_g$ = gyroscopic stability factor

and where the aerodynamic damping term $\varepsilon_A$ is - for the purposes of our program - a specified constant.

One of the program options, then, is the determination of $\varepsilon(\tau)$ from Eq. (8.1) for a specified $B_1$, $B_2$, $\varepsilon_A$ and range of $\tau$. Note that $\varepsilon$ appears implicitly on the right-hand side of Eq. (8.1); thus an indirect, iterative method of solving (8.1) for $\varepsilon$ is needed. For a fixed $\tau$, an $\varepsilon$ value is assumed; $C_{LSM}(\varepsilon,\tau)$ and $C_{LIM}(\varepsilon,\tau)$ are then computed as in Section V and a new $\varepsilon$ obtained from (8.1). The new $\varepsilon$ value replaces the old one and the process is repeated until (in a well-ordered universe) it converges on the proper $\varepsilon(\tau)$. This approach is carried out for a set of $\tau$ values over the specified range.

For most of the cases we have considered so far, $(s_g)^{-1}$ has been taken as zero; that is, $B_2 = 1$. The parameter $B_1$, however, is not necessarily a constant. The user has a choice: constant $I_x$ (and hence constant $B_1$) or constant transverse moment of inertia, $I_y$. In the latter case, the program computes the variable $I_x$ from the relation

$$I_x(\varepsilon,\tau) = \frac{\tau\, I_y}{B_2} - m_L a^2\, C_{LIM}(\varepsilon,\tau)\ . \tag{8.2}$$

To specify $B_1$, then, the user inputs:

1. $\rho_L$, a and c (to allow the program to compute the constant mass $m_L$ from Eq. (2.5));

2. either $I_x$ or $I_y$ (whichever is constant).

## IX. THE PRESSURE COEFFICIENT

Murphy's complex pressure coefficient, $C_p \exp(i\phi_p)$, is a measure of the fluctuating part of the inviscid pressure. In Reference 1, he derives the following expression:

$$C_p\, e^{i\phi_p} = \left(\frac{c}{a}\right)\left[\, is\,(2+is)\left(\frac{r}{a}\right)\left(\frac{x}{c}\right) - S_1(r,x)\right] \tag{9.1}$$

where $S_1(r,x)$ is a summation defined in Eq. (A1) of Appendix A. The final optional task our program can perform is to compute the absolute value $C_p$ and the argument $\phi_p$ from Eq. (9.1).

There are various ways in which this could be done. For example:

1. Fix $\varepsilon$, r and x; compute $C_p$ and $\phi_p$ as functions of $\tau$.

2. Fix $\varepsilon$, $\tau$ and $r$; compute $C_p$ and $\phi_p$ as functions of $x$.

3. Fix $\varepsilon$, $\tau$ and $x$; compute $C_p$ and $\phi_p$ as functions of $r$.

So far we have had a need to encode only a special case of 1 above. Namely, for $x = c$, $\varepsilon = 0$ and any specified $r/a$, the program will compute

$$C_p\ (\tau)\ e^{i\phi_p(\tau)} = -\left(\frac{c}{a}\right)\left[\tau\ (2 - \tau)\ \left(\frac{r}{a}\right) + S_1\ (r,c)\right] \tag{9.2}$$

## X. SUMMARY

The program based on Murphy's equations has so far been used mainly to compute the liquid side moment coefficient $C_{LSM}$ and/or the eigenvalues $s_{kn}$.

For a specified Re, b/a, c/a, $\varepsilon$ and a range of $\tau$ values, $C_{LSM}$ is computed from Eqs. (5.1-5.4) and Definition (2.6). (The same equations yield $C_{LIM}$ but interest in this in-plane moment coefficient is low.)

For a specified Re, b/a, c/a and k, the eigenfrequencies $s_{kn}$ are computed for n = 1 - 6 from Eq. (7.1) and Definition (4.12).

# APPENDIX A. THE INVISCID PERTURBATION VARIABLES

In Reference 1, Murphy derives an expression for $p_{si}(r,x)$. We repeat that expression here and also give expressions for $u_{si}$, $v_{si}$, $w_{si}$ and their first partia' derivatives.

For convenience, we first define five sums:

$$S_1(r,x) = \sum R_k(r)\, X_k(x) \tag{A1}$$

$$S_2(r,x) = a \frac{\partial S_1}{\partial r} = a \sum R_k'(r)\, X_k(x) \tag{A2}$$

$$S_3(r,x) = c \frac{\partial S_1}{\partial x} = c \sum R_k(r)\, X_k'(x) \tag{A3}$$

$$S_4(r,x) = a \frac{\partial S_3}{\partial r} = ac \sum R_k'(r)\, X_k'(r) = c \frac{\partial S_2}{\partial x} \tag{A4}$$

$$S_5(r,x) = -c \frac{\partial S_3}{\partial x} = \sum R_k(r)\, X_k(x)\, \lambda_k^2 \tag{A5}$$

where $X_k'(x)$ follows from Eq. (3.6), $R_k'(r)$ is given in Eq. (4.14) and where the summations are for $k = 1, 3, 5, ...$

Then we have:

$$p_{si}(r,x) = -(c/a)\, \hat{K} \left[S_1 - (1 + is)^2 (x/c)\, (r/a)\right] \tag{A6}$$

$$u_{si}(r,x) = i\, \hat{K} \left[\frac{S_3}{1 + is} - (1 + is)\, (r/a)\right] \tag{A7}$$

$$v_{si}(r,x) = \frac{-i\, (c/a)\, \hat{K}}{1 - is} \left[\frac{(2\, a/r)\, S_1 + (1 + is)\, S_2}{3 + is} - (1 + is)^2 (x/c)\right] \tag{A8}$$

$$w_{si}(r,x) = \frac{-(c/a)\, \hat{K}}{1 - is} \left[\frac{(1 + is)\, (a/r)\, S_1 + 2\, S_2}{3 + is} - (1 + is)^2 (x/c)\right] \tag{A9}$$

$$w_{si} + i\, v_{si} = \frac{(c/a)\,\hat{K}}{3 + is} \left[(a/r)\, S_1 - S_2\right] \tag{A10}$$

$$w_{si} - i\, v_{si} = -\frac{(c/a)\,\hat{K}}{1 - is} \left[(a/r)\, S_1 + S_2 - 2\,(1 + is)^2\,(x/c)\right] . \tag{A11}$$

For the first partials with respect to r, we have:

$$a\,\frac{\partial p_{si}}{\partial r} = -\,(c/a)\,\hat{K}\left[S_2 - (1 + is)^2\,(x/c)\right] \tag{A12}$$

$$a\,\frac{\partial u_{si}}{\partial r} = i\,\hat{K}\left[\frac{S_4}{1 + is} - (1 + is)\right] \tag{A13}$$

$$a\,\frac{\partial v_{si}}{\partial r} = i\left(\frac{c}{a}\right)\hat{K}\left[\frac{(a/r)^2\, S_1 - (a/r)\, S_2}{3 + is} + \frac{(a/c)^2\, S_5}{1 + is}\right] \tag{A14}$$

$$a\,\frac{\partial w_{si}}{\partial r} = \left(\frac{c}{a}\right)\hat{K}\left[\frac{-(a/r)^2\, S_1 + (a/r)\, S_2}{3 + is} + \frac{2\,(a/c)^2\, S_5}{(1 + is)^2}\right] \tag{A15}$$

$$\frac{a\,\partial\,(w_{si} + i\, v_{si})}{\partial r} = \left(\frac{c}{a}\right)\hat{K}\left[\frac{-2\,(a/r)^2\, S_1 + 2\,(a/r)\, S_2}{3 + is} + \frac{(a/c)^2\,(1 - is)\, S_5}{(1 + is)^2}\right] \tag{A16}$$

$$\frac{a\,\partial\,(w_{si} - i\, v_{si})}{\partial r} = \frac{(a/c)\,(3 + is)\,\hat{K}\, S_5}{(1 + is)^2} . \tag{A17}$$

For the first partials with respect to x, we have:

$$c\,\frac{\partial p_{si}}{\partial x} = \left(\frac{c}{a}\right)\hat{K}\left[(1 + is)^2\left(\frac{r}{a}\right) - S_3\right] \tag{A18}$$

$$c\,\frac{\partial u_{si}}{\partial x} = -\,\frac{i\,\hat{K}\, S_5}{1 + is} \tag{A19}$$

$$c\,\frac{\partial v_{si}}{\partial x} = \frac{i\,(c/a)\,\hat{K}}{1 - is}\left[(1 + is)^2 - \frac{(2\,a/r)\,S_3 + (1 + is)\,S_4}{3 + is}\right] \tag{A20}$$

$$c\,\frac{\partial w_{si}}{\partial x} = \frac{(c/a)\,\hat{K}}{1 - is}\left[(1 + is)^2 - \frac{(1 + is)\,(a/r)\,S_3 + 2\,S_4}{3 + is}\right] \tag{A21}$$

$$c\,\frac{\partial\,(w_{si} + i\,v_{si})}{\partial x} = \frac{(c/a)\,\hat{K}}{3 + is}\left[(a/r)\,S_3 - S_4\right] \tag{A22}$$

$$c\,\frac{\partial\,(w_{si} - i\,v_{si})}{\partial x} = \frac{(c/a)\,\hat{K}}{1 - is}\left[2\,(1 + is)^2 - (a/r)\,S_3 - S_4\right] \quad . \tag{A23}$$

## APPENDIX B. VISCOUS PERTURBATION VARIABLES AT THE LATERAL WALL, $r = a$

In Reference 1, Murphy gives the following lateral wall boundary conditions:

$$p_{sv}(a,x) = 2\ \delta_a\ w_{sv}(a,x) \tag{B1}$$

$$u_{sv}(a,x) = -u_{si}(a,x) - i\ (1 + is)\ \hat{K} \tag{B2}$$

$$v_{sv}(a,x) = \delta_a \left\{ -v_{si}(a,x) - \left[ a \frac{\partial v_{si}}{\partial r} \right]_{r = a} + i\ (1 + is)\ (x/a)\ \hat{K} \right\} \tag{B3}$$

$$w_{sv}(a,x) = -\ w_{si}(a,x) + (1 + is)\ (x/a)\ \hat{K} \quad . \tag{B4}$$

At $r = a$ and for $\delta_a \neq 0$, differentiation of these variables with respect to $r$ is equivalent to division by $\delta_a$:

$$\left[ \frac{a\ \partial p_{sv}}{\partial r} \right]_{r = a} = 2\ w_{sv}(a,x) \tag{B5}$$

$$\left[ \frac{a\ \partial u_{sv}}{\partial r} \right]_{r = a} = \frac{u_{sv}(a,x)}{\delta_a} \tag{B6}$$

$$\left[ \frac{a\ \partial v_{sv}}{\partial r} \right]_{r = a} = \frac{v_{sv}(a,x)}{\delta_a} \tag{B7}$$

$$\left[ \frac{a\ \partial w_{sv}}{\partial r} \right]_{r = a} = \frac{w_{sv}(a,x)}{\delta_a} \quad . \tag{B8}$$

## APPENDIX C. VISCOUS PERTURBATION VARIABLES AT THE END WALLS, $x = \pm c$

For $h = 0$, Murphy[1] gives or implies the following end-wall boundary conditions:

$$p_{sv}(r,c) = 0 \tag{C1}$$

$$u_{sv}(r,c) = -\delta_c \left[ c \frac{\partial u_{si}}{\partial x} \right]_{x = c} \tag{C2}$$

$$v_{sv}(r,c) = -v_{si}(r,c) + i\ (1 + is)\ (c/a)\ \hat{K} \tag{C3}$$

$$w_{sv}(r,c) = -w_{si}(r,c) + (1 + is)\ (c/a)\ \hat{K} \tag{C4}$$

and for the other end wall:

$$\left.\begin{aligned} p_{sv}(r,-c) &= 0 \\ u_{sv}(r,-c) &= +\ u_{sv}(r,c) \\ v_{sv}(r,-c) &= -\ v_{sv}(r,c) \\ w_{sv}(r,-c) &= -\ w_{sv}(r,c) \end{aligned}\right\} \tag{C5}$$

The following partial derivatives are given or implied in Reference 1:

$$\left[ c \frac{\partial p_{sv}}{\partial x} \right]_{x = c} = 0 \tag{C6}$$

$$\left[c\,\frac{\partial u_{sv}}{\partial x}\right]_{x=c} = -\left[c\,\frac{\partial u_{si}}{\partial x}\right]_{x=c} \tag{C7}$$

$$\left.\begin{aligned}\left[c\,\frac{\partial v_{sv}}{\partial x}\right]_{x=c} = {} & (i/2)\ (\alpha - \beta)\ w_{si}(r,c) \\ & - (1/2)\ (\alpha + \beta)\ v_{si}(r,c) \\ & + i\ (1 + is)\ (c/a)\ \hat{K}\ \beta\end{aligned}\right\} \tag{C8}$$

$$\left.\begin{aligned}\left[c\,\frac{\partial w_{sv}}{\partial x}\right]_{x=c} = {} & - (1/2)\ (\alpha + \beta)\ w_{si}(r,c) \\ & - (i/2)\ (\alpha - \beta)\ v_{si}(r,c) \\ & + (1 + is)\ (c/a)\ \hat{K}\ \beta\end{aligned}\right\} \tag{C9}$$

$$\left.\begin{aligned}\left[c\,\frac{\partial (w_{sv} - i\ v_{sv})}{\partial x}\right]_{x=c} = {} & - \beta\,[w_{si}(r,c) - i\ v_{si}(r,c)] \\ & + 2\ (1 + is)\ (c/a)\ \hat{K}\ \beta\end{aligned}\right\} \tag{C10}$$

where

$$\alpha = (c/a)\ [-i\ (3 + is)\ Re]^{1/2} \tag{C11}$$

$$\beta = (c/a)\ [i\ (1 - is)\ Re]^{1/2} \tag{C12}$$

and where the square roots are such that the real parts of $\alpha$ and $\beta$ are positive.

## APPENDIX D. DERIVATION OF THE TRANSVERSE LIQUID MOMENT COEFFICIENT EQUATIONS

Our final expression for $(C_{LM})_{p\ell}$, Eq. (5.1), is obtained by substituting in Eq. (2.9):

for $p_{si}(a,x)$, using Eq. (A6);

for $p_{sv}(a,x)$, using Eqs. (B1), (B4), and (A9).

The resulting integrand involves $xS_1(a,x)$ and $xS_2(a,x)$. Integration follows from the definition of $b_k$:

$$c^{-2} \int_{-c}^{c} x\ S_1(a,x)\ dx = T_1 \tag{D1}$$

$$c^{-2} \int_{-c}^{c} x\ S_2(a,x)\ dx = T_2 \quad . \tag{D2}$$

Our final expression for $(C_{LM})_{pe}$, Eq. (5.2), is obtained by substituting for $p_{si}(r,c)$ in Eq. (2.10), using Eq. (A6). The resulting integrand involves $r^2S_1(r,c)$. Integration follows from the properties of Bessel functions:

$$a^{-3} \int_{b}^{a} r^2\ S_1(r,c)\ dr = (c/a)^2\ [T_3 - (b/a)\ T_{3b}] \quad . \tag{D3}$$

Our final expression for $(C_{LM})_{v\ell}$, Eq. (5.3), is obtained by substituting in Eq. (2.11):

for $\left[a\ \frac{\partial w_{sv}}{\partial r}\right]_{r=a}$ , using Eqs. (B8), (B4), and (A9);

for $\left[a\ \frac{\partial u_{sv}}{\partial r}\right]_{r=a}$ , using Eqs. (B6), (B2), and (A7).

The resulting integrand involves $S_3(a,x)$. Integration follows at once from the definition of $S_3$:

$$c^{-1} \int_{-c}^{c} S_3(a,x)\, dx = 2T_4 \quad . \tag{D4}$$

Our final expression for $(C_{LM})_{ve}$, Eq. (5.4), is obtained by substituting for the partial derivative term in Eq. (2.12), using Eqs. (C10) and (A11). The resulting integrand involves the combination

$$aS_1(r,c) + rS_2(r,c) = a\sum[rR_k(r)]'\, X_k(c) \quad . \tag{D5}$$

Integration follows at once:

$$a^{-2} \int_{b}^{a} [a\, S_1(r,c) + r\, S_2(r,c)]\, dr = T_4 - (b/a)\, T_{4b} \quad . \tag{D6}$$

## APPENDIX E. NONZERO AXIAL OFFSET

Here, we consider the case where the center of mass of the liquid-filled cylinder is located a distance h forward of the projectile's center of mass.

The functions $R_k(r)$ were defined for k = 1, 3, 5,... by Eq. (4.1). For nonzero h, an $R_0$ must also be considered:

$$R_0(r) = \left(\frac{h}{c}\right)\left[\frac{E_0 r}{a} + \frac{F_0 a}{r}\right] \tag{E1}$$

where $E_0$, $F_0$, and $G_0$ have the same form as in Eqs. (4.10-4.12):

$$E_0 = \frac{c_1 - (c_{12}/c_{22})\, c_2}{G_0} \tag{E2}$$

$$F_0 = \frac{(c_{11}/c_{22})\, c_2 - (c_{21}/c_{22})\, c_1}{G_0} \tag{E3}$$

$$G_0 = c_{11} - (c_{12}/c_{22})\, c_{21} \quad . \tag{E4}$$

The definitions of $c_1$ and $c_2$ [Eqs. (4.7-4.8)] are unchanged, but the $c_{ij}$'s reduce to:

$$c_{11} = 3 + is \tag{E5}$$

$$c_{12} = (1 + 2\delta_a)\,(1 - is) \tag{E6}$$

$$c_{21} = (b/a)\,(3 + is)\, s^2 \tag{E7}$$

$$c_{22} = (b/a)^{-1}\,(1 - is)\,(2 + 4is - s^2) \quad . \tag{E8}$$

Note that for a fully-filled cylinder (b = 0), Eqs. (E2, E3) reduce to

$$\left.\begin{aligned} E_0 &= 2\, is\,(1 + is) \\ F_0 &= 0 \end{aligned}\right\} \qquad b = 0 \quad . \tag{E9}$$

In Reference 1, Murphy shows that the effect of h on the $(C_{LM})_{p\ell}$ equation is to add a small term:

$$(C_{LM})_{p\ell} = [\text{RHS of Eq. (2.9)}] + H_{p\ell} \tag{E10}$$

where

$$H_{p\ell} = -\frac{ih}{2a^2\tau} \int_{h-c}^{h+c} C_{po}(a)\, dx \tag{E11}$$

and where

$$C_{po}(a) = \left[R_0(r) - \frac{is\,(2 + is)\, hr}{ac}\right]_{r = a}$$

$$= \left(\frac{h}{c}\right)\left[E_0 + F_0 - is\,(2 + is)\right] \quad . \tag{E12}$$

Hence, we have

$$H_{p\ell} = -\frac{i\,(h/a)^2}{\tau}\,[E_0 + F_0 - is\,(2 + is)] \quad . \tag{E13}$$

For b = 0, this reduces to

$$H_{p\ell} = \frac{is^2\,(h/a)^2}{\tau} \qquad [b = 0] \tag{E13a}$$

and for $b = \varepsilon = 0$,

$$H_{p\ell} = -i\tau\,(h/a)^2 \qquad [b = \varepsilon = 0] \quad . \tag{E13b}$$

Offset h has no effect on $(C_{LM})_{pe}$ or on $(C_{LM})_{v\ell}$. The effect on the remaining component of $C_{LM}$ can, according to Murphy, be written as

$$(C_{LM})_{ve} = [\text{RHS of Eq. (2.12)}] + H_{ve} \tag{E14}$$

where

$$H_{ve} = \frac{ha}{2\,c^2\,\text{Re}\,\tau} \int_b^a \left[\frac{c}{K}\;\frac{\partial\,(w_{sv} - iv_{sv})}{\partial x}\right]_{x = h - c}^{x = h + c} \frac{r\,dr}{a^2} \quad . \tag{E15}$$

From relations given by Murphy, we can simplify the integrand:

$$\left[ \frac{c}{\hat{K}} \quad \frac{\partial (w_{sv} - iv_{sv})}{\partial x} \right]_{x = h - c}^{x = h + c} = \frac{4\beta (h/a) [E_0 - 2is (1 + is)]}{1 - is} \tag{E16}$$

where $\beta$ is defined in Eq. (C12) and where

$$\beta \, Re^{-1} = (c/a) (1 + i) \left( \frac{1 - is}{2 Re} \right)^{1/2} \quad . \tag{E17}$$

Substituting (E16) and (E17) in (E15), we have

$$H_{ve} = (h/c)^2 (c/a) \left( 1 - \frac{b^2}{a^2} \right) \frac{(1 + i) [E_0 - 2is (1 + is)]}{\tau [2 (1 - is) Re]^{1/2}} \quad . \tag{E18}$$

Thus, the effect of offset h on the liquid moment coefficients can be written as:

$$C_{LSM} = (C_{LSM})_{h=0} + R \{ H_{p\ell} + H_{ve} \} \tag{E19}$$

$$C_{LIM} = (C_{LIM})_{h=0} + I \{ H_{p\ell} + H_{ve} \} \quad . \tag{E20}$$

## APPENDIX F. CENTRAL ROD

If an air core of radius b is replaced by a central rod of radius d, the formulas for $E_k$ and $F_k$ change. System (4.4) becomes

$$\left.\begin{aligned} c_{11}\ E_k + c_{12}\ F_k &= c_1 \\ \hat{c}_{11}\ E_k + \hat{c}_{12}\ F_k &= (d/a)\ c_1 \end{aligned}\right\} \qquad \text{(F1)}$$

where

$$\begin{aligned} \hat{c}_{11} = {} & [(1 - 2\ \hat{\delta}_a)(1 - is) - (1 + is)\ A_{kd}^2\ \hat{\delta}_a]\ J_{1d} \\ & + [1 + is + (1 - is)\ \hat{\delta}_a]\ A_{kd}\ J_{0d} \end{aligned} \qquad \text{(F2)}$$

$\hat{c}_{12}$ = form identical to $\hat{c}_{11}$ with J replaced by Y

and where*

$$\left.\begin{aligned} \hat{\delta}_a &= a\ \delta_a\ (d + a\ \delta_a)^{-1} \\ A_{kd} &= A_k\ (d) = \hat{\lambda}_k\ d/c \\ J_{0d} &= J_0\ (A_{kd}) \\ J_{1d} &= J_1\ (A_{kd}) \end{aligned}\right\} \qquad \text{(F3)}$$

and similarly for $Y_{0d}$, $Y_{1d}$. The rodded $E_k$ and $F_k$ values are the solutions of system (F1), the rodded determinant $G_k$ is given by

$$(G_k)_{rod} = c_{11}\ \hat{c}_{12} - \hat{c}_{11}\ c_{12} \qquad \text{(F4)}$$

and the rodded eigenvalues are the roots of the equation

$$(G_k)_{rod} = 0. \qquad \text{(F5)}$$

---

* *In Reference 1, Murphy approximates $\hat{\delta}_a$ by $a\,\delta_a/d$ in $\hat{c}_{11}$ and $\hat{c}_{12}$. This is consistent with his use of $\delta_a$ for $\tilde{\delta}_a$ in Eq. (4.5) but is less valid. The distinction between $(d + a\ \delta_a)^{-1}$ and $d^{-1}$ can be significant for small rod radius d.*

The rodded value of $C_{LM}$ will depend, of course, on the $E_k$'s and $F_k$'s computed from Eq. (F1) rather than Eq. (4.4). In addition, $C_{LM}$ is affected by explicit changes in Eqs. (5.1-5.4). For the two end-wall components, the change consists of replacing b with d:

$$\left[(C_{LM})_{pe}\right]_{rod} = \text{the } (C_{LM})_{pe} \text{ of Eq. (5.2) with b replaced by d and } T_{3b} \text{ replaced by } T_{3d} \tag{F6}$$

$$\left[(C_{LM})_{ve}\right]_{rod} = \text{the } (C_{LM})_{ve} \text{ of Eq. (5.4) with b replaced by d and } T_{4b} \text{ replaced by } T_{4d} \tag{F7}$$

where

$$T_{3d} = \sum \left[R_k(d) - dR_k'(d)\right] \cdot \hat{\lambda}_k^{-2} \cdot X_k(c) \tag{F8}$$

$$T_{4d} = \sum R_k(d) \cdot X_k(c) \quad . \tag{F9}$$

For the two lateral components, new terms appear in Eqs. (5.1) and (5.3):

$$\left[(C_{LM})_{p\ell}\right]_{rod} = \frac{i\ (c/a)^2}{\tau}\left\{\left(1 - \frac{d^2}{a^2}\right)\frac{is\ (2 + is)}{3} - \frac{1}{2}\left[T_1 - (d/a)\ T_{1d}\right] + \delta_a\left[T_5 + T_{5d}\right]\right\} \tag{F10}$$

$$\left[(C_{LM})_{v\ell}\right]_{rod} = -\frac{i\ \delta_a}{\tau}\left\{T_4 + (d/a)^2\ T_{4d} + \frac{(c/a)^2\ (1 + is)}{2}\left[T_5 + T_{5d}\right]\right\} \tag{F11}$$

where

$$T_{1d} = \sum R_k(d) \cdot \bar{b}_k \tag{F12}$$

$$T_{2d} = \sum d\, R_k{}' \,(d) \cdot \bar{b}_k \tag{F13}$$

$$T_{5d} = \frac{1}{1 - is} \left[ \frac{(1 + is)\, T_{1d} + 2\, T_{2d}}{3 + is} - \frac{4\,(d/a)\, is\, (1 + is)}{3} \right] . \tag{F14}$$

## LIST OF SYMBOLS

| | | |
|---|---|---|
| $a$ | radius of the liquid-payload cylinder | |
| $a_k$ | coefficients in the least-squares fit: $\sum a_k X_k(x) = \frac{x}{c}$ | [k = 1,3,5,...N] |
| $b$ | radius of the air core in a partially filled cylinder | |
| $b_k$ | $c^{-2} \int_{-c}^{c} \bar{X}_k(x)\, x\, dx$ | [k = 1,3,5,...] |
| $b_{jk}$ | $c^{-1} \int_{-c}^{c} \bar{X}_j(x)\, X_k(x)\, dx$ | [j, k = 1,3,5,...] |
| $c$ | half-height of the liquid-payload cylinder | |
| $d$ | radius of a central rod within the liquid-payload cylinder | |
| $h$ | axial offset: the distance between the centers of mass of the projectile and its liquid-payload cylinder | |
| $k$ | axial wave number | [1,3,5,...] |
| $m$ | azimuthal wave number (taken as 1 in this report) | |
| $m_L$ | $2\pi a^2 c\, \rho_L$, the mass of the liquid in a fully filled cylinder | |
| $n$ | radial wave number | [1,2,3,...] |
| $p_s$ | nondimensional pressure perturbation | |
| $p_{si}$ | inviscid component of $p_s$ | |
| $p_{sv}$ | viscous component of $p_s$ | |
| $r$ | radial coordinate in an earth-fixed cylindrical system | |
| $s$ | $(\varepsilon + i)\,\tau$ | |

| | |
|---|---|
| $s_g$ | gyroscopic stability factor |
| $s_{kn}$ | eigenvalue of s for wave numbers k and n |
| t | time |
| $u_s, v_s, w_s$ | axial, radial and azimuthal components of the nondimensional velocity perturbation |
| $u_{si}, v_{si}, w_{si}$ | inviscid components of $u_s$, $v_s$, $w_s$ |
| $u_{sv}, v_{sv}, w_{sv}$ | viscous components of $u_s$, $v_s$, $w_s$ |
| x | axial coordinate in an earth-fixed cylindrical system |
| $A_k(r)$ | $\hat{\lambda}_k r/c$, the argument of the Bessel functions |
| $B_1$ | $m_L a^2/I_x$ |
| $B_2$ | $1 - (4\ s_g)^{-1}$ |
| $C_{LIM}$ | liquid in-plane moment coefficient, Eqs. (2.4, 2.6) |
| $C_{LM}$ | $C_{LSM} + i\ C_{LIM}$, transverse liquid moment coefficient, Eq. (2.4) |
| $(C_{LM})_{pe}$ | that part of $C_{LM}$ due to pressure on the end walls of the cylinder |
| $(C_{LM})_{p\ell}$ | that part of $C_{LM}$ due to pressure on the lateral wall of the cylinder |
| $(C_{LM})_{ve}$ | that part of $C_{LM}$ due to viscous shear on the end walls of the cylinder |
| $(C_{LM})_{v\ell}$ | that part of $C_{LM}$ due to viscous shear on the lateral wall of the cylinder |

| | |
|---|---|
| $C_{LRM}$ | liquid roll moment coefficient, Eq. (6.2) |
| $C_{LSM}$ | liquid side moment coefficient, Eqs. (2.4, 2.6) |
| $C_p$ | nondimensional magnitude of the complex pressure coefficient, Eq. (9.1) |
| $E_0(s)$, $F_0(s)$ | complex functions in the definition of $R_0$; computed by Eqs. (E2 - E3) |
| $E_k(s)$, $F_k(s)$ | complex functions in the definition of $R_k$; computed by Eqs. (4.10 - 4.11) $[k = 1,3,5,...]$ |
| $G_0(s)$ | determinant of the system that determines $E_0$ and $F_0$, Eq. (E4) |
| $G_k(s)$ | determinant of the system (4.4) that determines $E_k$ and $F_k$; the roots of $G_k(s) = 0$ are the eigenvalues $[k = 1,3,5,...]$ |
| $I_x$, $I_y$ | axial and transverse moments of inertia of the projectile |
| $J_0$, $J_1$ | Bessel functions of the first kind of order 0 and 1 |
| $\hat{K}$ | complex constant in the definition of $\tilde{\xi}$, Eq. (2.1); Murphy's equations were obtained by linearizing with respect to $\hat{K}$ |
| $M_{LX}$, $M_{L\tilde{Y}}$, $M_{L\tilde{Z}}$ | rectangular components of the liquid moment in an aeroballistic non-rolling system |
| $N$ | arbitrary upper limit on k for computational purposes (taken as 29 in our program) |
| $R_0(r)$ | $\left(\frac{h}{c}\right)\left[\frac{E_0 r}{a} + \frac{F_0 a}{r}\right]$ |
| Re | Reynolds number, $\frac{\dot{\phi} a^2}{\nu}$ |
| $R_k(r)$ | $a_k [E_k J_1 (A_k) + F_k Y_1 (A_k)]$ $[k = 1,3,5,...]$ |

| | |
|---|---|
| $S_i$ | summations defined in Appendix A [$i = 1 - 5$] |
| $T_i$ | summations needed to compute $C_{LSM}$ and $C_{LIM}$, Eqs. (5.5 - 5.11) |
| $X_k(x)$ | $\sin(\lambda_k x/c)$ [$k = 1,3,5,\ldots$] |
| $Y_0$, $Y_1$ | Bessel functions of the second kind of order 0 and 1 |
| $\alpha$ | $(c/a)\,[-\,i\,(3 + is)\,Re]^{1/2}$ |
| $\beta$ | $(c/a)\,[i\,(1 - is)\,Re]^{1/2}$ |
| $\delta_a$ | $(1 + i)\,[2\,(1 + is)\,Re]^{-1/2}$ |
| $\delta_c$ | $\dfrac{1}{2\,(1 + is)} \left[\dfrac{3 + is}{\beta} - \dfrac{1 - is}{\alpha}\right]$ |
| $\epsilon$ | (yaw damping rate)/(coning rate) |
| $\epsilon_A$ | aerodynamic damping constant, Eq. (8.1) |
| $\theta$ | azimuthal coordinate in an earth-fixed cylindrical system |
| $\lambda_k$ | nondimensional frequency number, determined from Eq. (3.2) [$k = 1,3,5,\ldots$] |
| $\hat{\lambda}_k$ | $\dfrac{[(3 + is)\,(1 - is)]^{1/2}\,\lambda_k}{1 + is}$ |
| $\nu$ | kinematic viscosity of the liquid |

## LIST OF SYMBOLS (Continued)

| | |
|---|---|
| $\tilde{\xi}$ | $\hat{K}\, e^{s\phi}$, the complex yaw in an aeroballistic non-rolling coordinate system |
| $\rho_L$ | density of the liquid |
| $\tau$ | coning rate/$\dot{\phi}$ |
| $\tau_{kn}$ | eigenvalue of $\tau$ for wave numbers k and n |
| $\phi$ | $\dot{\phi}t$ |
| $\dot{\phi}$ | spin (assumed positive and constant) |
| $\phi_p$ | orientation angle of the complex pressure coefficient, Eq. (9.1) |
| $[\bar{\ }]$ | complex conjugate |
| $[\ ]'$ | derivative with respect to whatever independent variable is present |
| $[\ ]_b$ | value at $r = b$ |
| $[\ ]_d$ | value at $r = d$ |

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